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ASSIGNMENT NO. 1

QUESTION NO 1: (a) The Metrology Department has collected the following data for snowfall in Murree during the month of January.

No of Snowfall	0	1	2	3	4	5	6	7	8	9	More than 9
Probability	0.02	0.04	0.06	0.02	0.05	0.10	0.13	0.20	0.25	0.08	0.05

What is the probability that there will be 3 or more snowfalls this January?

Answer:

$$P \leq 3 = (P = 3) + (P = 4) + (P = 5) + (P = 6) \\ + (P = 7) + (P = 8) + (P = 9) \\ + (P = \text{more than } 9)$$

$$= 0.02 + 0.05 + 0.1 + 0.13 + 0.20 + 0.25 + 0.08 + 0.05 \\ = 0.88$$

So there is 88% probability that there will be 3 or more snowfalls this January.

ii. What is the probability that there will be at max 6 snowfalls this January?

Answer:

The probability of maximum 6 snowfalls will be equal to the sum of probabilities from 0 to 6.

$$\begin{aligned} P_{\max 6} &= (P=0) + (P=1) + (P=2) + (P=3) + (P=4) + (P=5) + (P=6) \\ &= 0.02 + 0.04 + 0.06 + 0.02 + 0.05 + 0.10 + 0.13 \\ &= 0.42 \end{aligned}$$

The probability of maximum 6 snowfalls will be 42%.

(b) Given the card are selected at random, without replacement, from a standard 52-card deck, determine the probability that;

i. The first 2 cards are hearts

Answer:

There are total 52 cards in 9 decks. In these 52 cards 13 are hearts, 13 are clubs, 13 are spades and 13 are diamonds. The probability of heart is $13/52$. In 2nd attempt without replacement the probability of coming another heart is $12/51$. This is due to the reason that after one card is withdrawn, 51 cards will leave behind. So, the probability of coming another heart will be $12/51$. In this case the probability of first two cards are heart will become $P_{2n} = P_1, P_2$

$$\begin{aligned} P_{2n} &= 13/52 * 12/51 \\ &= 156/2704 \\ &= 0.058 \\ &= 5.8\% \end{aligned}$$

ii. The first is a spade, the second a club, the third a heart, and the fourth a diamond

Answer:

The probability of spade to appear first will be $13/52$

$$\begin{aligned}P_s &= 13/52 \\ &= 0.25 \\ &= 25\%\end{aligned}$$

Now, 51 cards will be left behind. The probability of appearing club will be 13/51.

$$\begin{aligned}P_c &= 13/51 \\ &= 0.254 \\ &= 25.4\%\end{aligned}$$

50 cards will be left behind probability of appearing heart will be 13/50.

$$\begin{aligned}P_h &= 13/50 \\ &= 0.26 \\ &= 26\%\end{aligned}$$

49 cards will be left behind. Probability of appearing diamond will be 13/49.

$$\begin{aligned}P_d &= 13/49 \\ &= 0.265 \\ &= 26.5\%\end{aligned}$$

In order to find the probability of finding a spade first, a club second, a heart third and a diamond fourth, we have to multiply all probabilities given above.

$$\begin{aligned}P_{scnd} &= (13/52) * (13/51) * (13/50) * (13/49) \\ &= 2197 / 499800 \\ &= 0.0044 \\ &= 0.44\%\end{aligned}$$

iii. 3 aces are selected in a row

Probability of coming ace in 1st, 2nd, 3rd and 4th attempt will be 4/52, 4/51, 4/50 and 4/49 without replacement. Now if we take it in a reverse direction than we can find the probability of no aces in first 4 cards without.

Probability of coming no ace in 1st, 2nd, 3rd and 4th attempt will be 48/52 or 12/13, 47/51, 46/50 or 23/25 and 45/49. If we multiply these four probabilities than we can find out the probability of

$$\begin{aligned} P_{n4a} &= 4,669,920 / 6,497,400 \\ &= 0.719 \\ &= 71.9\% \end{aligned}$$

QUESTION NO 2:

The sample space of an experiment consists of five simple events: E_1, E_2, E_3, E_4 and E_5 . These events are mutually exclusive. The probabilities of occurrence of these events are $P(E_1) = .20, P(E_2) = .15, P(E_3) = .25, P(E_4) = .30, P(E_5) = .10$. Several compound events can be defined for this experiment. They are: $F = \{E_1, E_2, E_3\}, G = \{E_1, E_3, E_5\}, H = \{E_4, E_5\}$. Determine.

(i) $P(G')$

Answer:

$$\begin{aligned} P(G') &= 1 - P(G) \\ P(G') &= 1 - [P(E_1) + P(E_3) + P(E_5)] \\ P(G') &= 1 - (0.20 + 0.25 + 0.10) \\ P(G') &= 1 - 0.55 \\ P(G') &= 0.45 \end{aligned}$$

(ii) $P(F \cup G)$

Answer:

$$\begin{aligned} P(F \cup G) &= \\ F \cup G &= \{E_1, E_2, E_3\} \cup \{E_1, E_3, E_5\} \\ P(F \cup G) &= P(E_1) + P(E_2) + P(E_3) + P(E_5) \\ P(F \cup G) &= 0.20 + 0.15 + 0.25 + 0.10 \\ P(F \cup G) &= 0.70 \end{aligned}$$

(iii) $P(G \cup H)$

Answer:

$$\begin{aligned} P(G \cup H) &= \\ G \cup H &= \{E_1, E_3, E_5\} \cup \{E_4, E_5\} \\ G \cup H &= \{E_1, E_3, E_4, E_5\} \end{aligned}$$

$$P(G \cup H) = P(E_1) + P(E_3) + P(E_4) + P(E_5)$$

$$P(G \cup H) = 0.20 + 0.25 + 0.30 + 0.10$$

$$P(G \cup H) = 0.85$$

(iv) $P(F \cap H)$

Answer:

$$P(F \cap H) =$$

$$F \cap H = \{E_1, E_2, E_3\} \cap \{E_4, E_5\}$$

$$F \cap H = 0$$

\Rightarrow

$$P(F \cap H) = 0$$

(v) $P(F \cap G)$

Answer:

$$P(F \cap G) =$$

$$F \cap G = \{E_1, E_2, E_3\} \cap \{E_1, E_3, E_5\}$$

$$P(F \cap G) = P(E_1) + P(E_3)$$

$$P(F \cap G) = 0.20 + 0.25$$

$$P(F \cap G) = 0.45$$

(b) Five cards are selected at random from a deck of 52. If the drawn cards are not replaced in the deck, what is the probability of selecting an ace, king, ace, jack and ace, in that order?

Answer:

We know that there are four aces, four kings and four jacks present in a deck of 52 cards.

1. The probability of an ace selection is 1st attempt is 4/52.
2. The probability of an ace selection in 2nd attempt will be 4/51.
3. The probability of an ace selection in 3rd attempt will be 3/50 as one ace will already selected.
4. The probability of a jack selection is 4th attempt will be 4/49.
5. Finally the probability of an ace selection of fifth attempt will be 2/48.

Now if we want to calculate the probability of ace, king, ace, jack and are in that order we have to multiply the above given probabilities.

$$P_{akaja} = (4/52) * (4/51) * (3/50) * (4/49) * (2/48)$$

$$= 384 / 311,875,200 \Rightarrow 0.00000123 \Rightarrow P_{akaja} = 0.000123\%$$

QUESTION NO 3: Sketch the plane representing $3x + 0y + 8z = 5$

Answer:

We first sketch 3 traces having two variables in them then combine these 3 traces to sketch a final plane

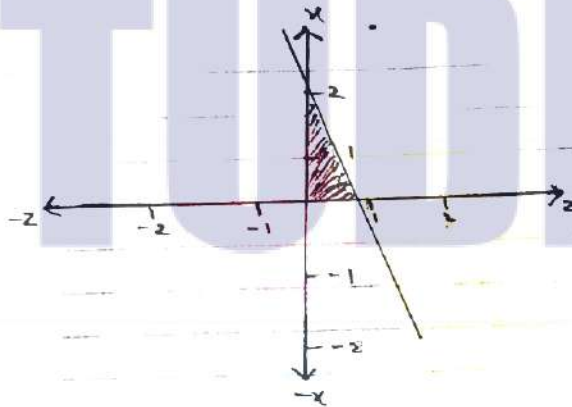
(i) $3x + 0y + 8z = 5$

For x intercept put $z = 0$

$$3x = 5 \Rightarrow x = 5/3 \Rightarrow x = 1.66$$

for z intercept put $x = 0$

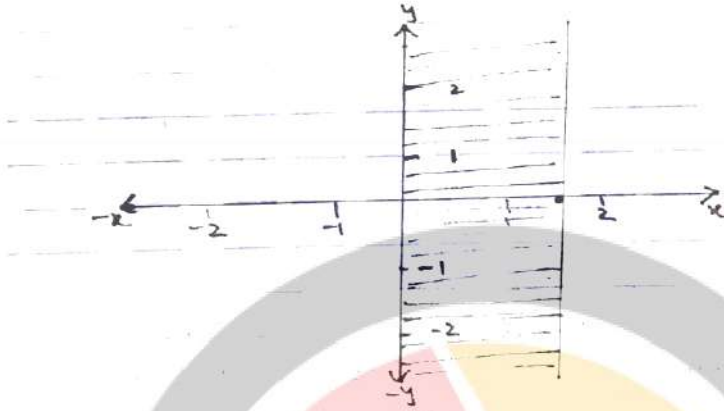
$$8z = 5 \Rightarrow z = 5/8 \Rightarrow z = 0.625$$



(ii) $3x + 0y = 5$

After simplifying this equation $3x=5$ So, x-intercept will be equal to 1.66

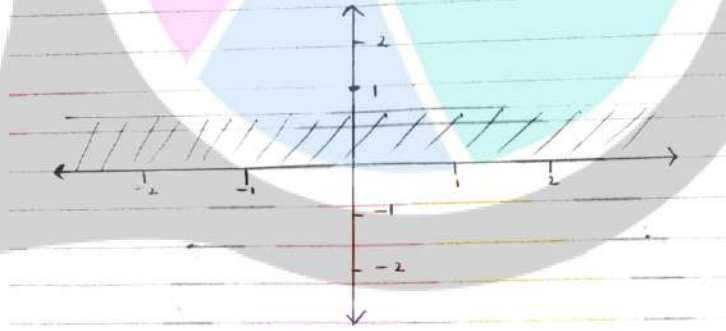
$$X = 1.66$$



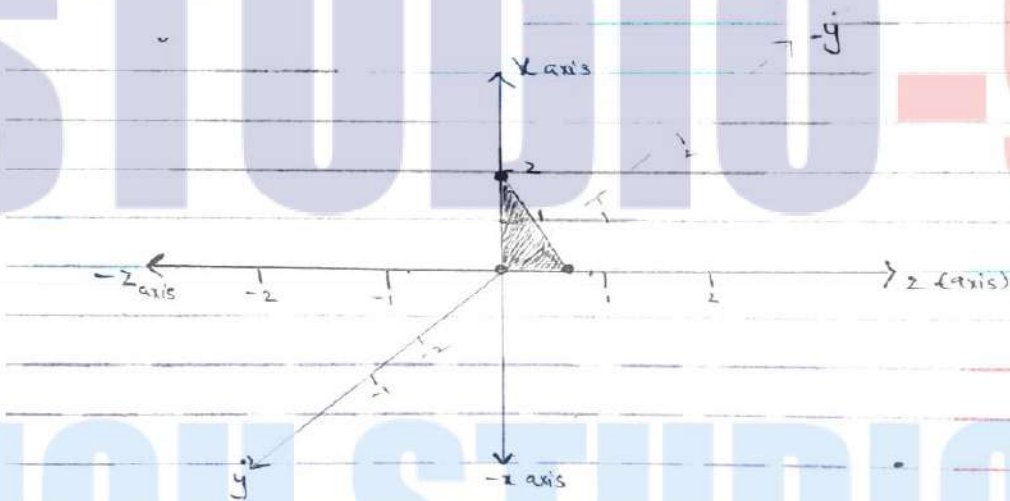
(iii) $0y + 8z = 5$

After simplifying this equation

$Z = 5/8 \Rightarrow Z = 0.625$



After combining all 3 planes given above we will get the sketch of the plane.



(b) Give the points $(-4, 8)$ and $(6, -12)$:

(i) Determine the midpoint of the line segment connecting the points.

Answer:

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Let $A = (-4,8)$ and $B = (6,12)$

Mid point of $AB = M$

$$M = ((x_1 + x_2)/2, (y_1 + y_2)/2)$$

Here $x_1 = -4$, $y_1 = 8$, $x_2 = 6$ and $y_2 = -12$

Putting these values in above given formula

$$M = ((-4+6)/2, (8-12)/2) \Rightarrow M = (2/2, -4/2)$$

$M = (1, -2) \Rightarrow$ So the midpoint of AB is $(1, -2)$

(ii) Determine the distance separating the two points.

Ans.

$A = (-4,8)$ and $B (6,12)$

Here $x_1 = -4$, $y_1 = 8$, $x_2 = 6$ and $y_2 = -12$

We know that

$$d = |AB| = \sqrt{((x_2 - x_1)^2 + (y_2 - y_1)^2)}$$

$$d = |AB| = \sqrt{((6 - (-4))^2 + ((-12 - 8))^2)}$$

$$d = |AB| = \sqrt{(6+4)^2 + (-20)^2}$$

$$d = |AB| = \sqrt{500} \Rightarrow d = |AB| = 10\sqrt{5}$$

QUESTION NO 4:

Explain the difference between point-slope form and slope-intercept form of a linear equation and the conversion between them.

Answer:

Modifying an equation of certain type into another is useful for us in certain case. This provides a lot of useful information. This procedure

is similar to the procedure of making toys of different shapes with the help of clay. Sometimes we have to change the equation to extract the information we need.

Slope Intercept Form

The slope intercept form of an equation is

$$Y = mx + b$$

About b

Here 'b' tells about y-intercept. This 'b' represents a point where a line passes y-axis.

About m:

Here m tells us about the slope of given linear equation. This represents a line.

Purpose of Using of Slope Intercept Form

Normally we use slope intercept form to find slope and y intercept for a given line. In other words slope intercept form is an easy way to write an equation of line in such a way that we will be able to find its slope and y intercept just after looking at the equation.

Let us understand the above statements with the help of an example,

Example:

Find the equation of line that is passing through the given two points (-4,-6) and (10,6). And write it in slope intercept form

We know that

$$Y = mx + b$$

We also know that

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

In above given points $x_1 = -4$, $y_1 = -6$, $x_2 = 10$ and $y_2 = 6$

$$m = \frac{10 - (-4)}{6 - (-6)} \Rightarrow m = \frac{14}{12} \Rightarrow m = \frac{7}{6}$$

$$y = \frac{7}{6}x + b$$

To find the value of b we have to put $x = -4$ and $y = -6$.

$$-6 = \frac{7}{6}(-4) + b \Rightarrow b = -6 + \frac{14}{3} \Rightarrow b = \frac{-18 - 14}{3} \Rightarrow b = \frac{-4}{3}$$

$$y = \frac{7}{6}x - \frac{4}{3}$$

Point Slope Form

Point Slope – form also called point – gradient form is out of three forms of equation that are used to describe a straight line. The advantage of this form is that we can write the equation of line after having only one point and slope,

Point – slope is written as: $y - y_1 = m(x - x_2)$

About x_1 and x_2

In above formula x_1 and x_2 are just the numbers that represent x-coordinate and y-coordinate respectively on given line.

Example:

Find the equation of line given that point (4,6) and slope=3

We know that $y - y_1 = m(x - x_2)$

$x_1 = 4$ and $y_2 = 6$ and $m=3$

Equation we get

$$y - 6 = 3(x - 4)$$

Conversion of Point-slope form In the Slope – Intercept form

We know that point – slope form is given by

$$y - y_1 = m(x - x_2)$$

We also know that slope – intercept form is given by $y = mx + b$

After looking at these two forms it is clear that both these form have 'm' common in them, if we have given a point and slope of line that we can write point – slope form easily. Once we have point and slope it will be easier for us to write slope intercept form as we have seen in example under slope – intercept form. So, form point –slope form we can easily write slope – intercept form. In other words we can convert point – slope form into slop – intercept form.

Let us understand the whole process with the help of example.

$$y-6 = 3(x-4)$$

This the point – slope form of equation. After rearranging it use can easily write it in slop – intercept form.

$$Y = 3x -12 +6$$

$$Y = 3x -6$$

M = 3 and y – intercept is -6. The above equation is slope intercept form of linear equation.

References:

Moloney, L (2020). How to convert point slope Form to slope intercept. Form <https://sciencing.com>

The Elbert Team (2021) slope – intercept form Explanation, review and examples

<https://www.albert.io>

- (b) If C equals degrees Celsius and F equals degrees Fahrenheit, assume that the relationship between the two temperature scales is linear and is being graphed with F on the vertical axis. Two data points on the line relating C and F are (5, 41) and (25, 77). Using these points, determine the slope-intercept equation which allows transformation from Celsius temperature to equivalent Fahrenheit temperature. Identify and interpret the meaning of the slope. C intercept, and F intercept.

Answer:

Given point (5,41) and (25,77)

First we find the value of slope.

$$m = \frac{77-41}{25-5}$$

$$m = \frac{36}{20}, m = \frac{9}{5}$$

Writing slope intercept form for F and C.

As mentioned in question F appears at vertical axis. So

$$F = \frac{9}{5}C + b$$

To find b put $F=41$ and $C = 5$

$$41 = \frac{9}{5} * 5 + b \Rightarrow 41 = 9 + b \Rightarrow b = 41 - 9 \Rightarrow b = 32$$

$$\text{Finally, } F = \frac{9}{5}C + 32$$

This is the slope – intercept form of this equation.

To find C – intercept we have to use $F = 0$

$$0 = \frac{9}{5}C + 32 \Rightarrow 0 = 9C + 160 \Rightarrow C = -160 / 9$$

Interpretation of Slope, y – intercept and x – intercept

Slope represents the rate of change of one quantity w.r.t other quantity. Here the slope of this equation tells us that for one Celsius change in temperature there will be 1.8 times increase in Fahrenheit temperature.

You may surprise to know that y – intercept in the above equation tells us about the melting point of ice / freezing point of water on Fahrenheit scale. The melting point of ice / freezing point of water is 32 F. it is 0 in Celsius scale C – intercept tells us about the value on Celsius scale at which value at Fahrenheit is zero.

QUESTION NO 5: (a) Solve the following second degree inequality:

(i) $x^2 + 4x - 12 \leq 0$

Answer:

By factorization

$$x^2 + 6x - 2x - 12 \leq 0$$

$$x(x+6) - 2(x+6) \leq 0$$

$$(x-2)(x+6) \leq 0$$

$$(x-2) \leq 0$$

Adding 2 on both sides ; Subtracting 6 on both sides

$$(x-2)+2 \leq 0+2$$

$$x \leq 2$$

Solution Set $S.S\{x: -6 \leq x \leq 2\}$

(ii) $5x^2 - 13x - 6 \geq 0$

Answer:

$$5x^2 - 13x - 6 \geq 0$$

$$5x^2 - 15x + 2x - 6 \geq 0$$

$$5x(x-3) + 2(x-3) \geq 0$$

$$(5x+2)(x-3) \geq 0$$

$$(5x+2) \geq 0$$

$$(x-3) \geq 0$$

Subtracting -2 on both

Adding 3 on both

Sides of inequality

sides of inequality

$$5x+2-2 \geq 0-2$$

$$x-3+3 \geq 0+3$$

$$5x \leq -2$$

$$x \geq 3$$

$$x \leq -2/5$$

Solution Set $S.S\{x: 3 \geq x \geq -2/5\}$

(b) Given the equation $\frac{x+y}{3} = 24 - x$

(i) Rewrite the equation in slope intercept form

Answer:

$$x+y = 72-3x$$

$$y = -4x + 72$$

We know that

$$y = mx + c$$

In this case

$$y = -4x + 72$$

(ii) Identify the slope and y- intercept

Answer:

$$y = -4x + 72$$

$$y = mx + c$$

After comparing above two equation

$$m = -4 \text{ and } c = 72$$

(i) Interpret the meaning of the slope

Answer:

Slope represents the rate of change of one quantity w.r.t quantity. Here the slope of this equation tells us about the rate of change of y w.r.t.x.

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